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## The role of the potassium inward rectifier in defining cell membrane potentials in low potassium media, analyzed by computer simulation

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#### Abstract

A model is presented that describes the contributions of the potassium conductance and the Na $^+$ /K $^+$  pump to the steady state magnitude of the plasma membrane potential,  $V_{\rm rest}$ , at different concentrations of potassium in the extracellular fluid:  $K_{\rm o}$ . The particular properties of the potassium inward rectifier, IKR, are shown to explain that  $V_{\rm rest}$  frequently depolarises on lowering  $K_{\rm o}$  below a critical value, because the IKR is only conductive when  $V_{\rm m}$  is near to the potassium equilibrium potential or more negative. The model aims at a generic description of the process based on compartmental analysis. It is worked out to describe  $V_{\rm rest}$  in mouse muscle fibres. Sensitivity analysis demonstrates that the process of switching off the IKR depends critically on the margin between  $V_{\rm rest}$  and  $E_{\rm K}$ , allowed for the IKR to remain open and the power of the Na/K pump to keep this margin small during the reduction of  $K_{\rm o}$ . However, cells exposed to  $K_{\rm o}$  just higher than the critical value may eventually also depolarize due to excessive loss of potassium. The model also demonstrates that loss of potassium membrane selectivity after blocking the Na/K pump by ouabain requires a mechanism additional to the above mentioned properties of the inward rectifier.

Key words: Computer simulation; Potassium; Inward Rectification; Na/K-pump; Membrane potential; Ouabain

#### 1. Introduction

In many cells at rest the dominant potassium membrane permeability is due to the presence of open inward rectifying potassium channels (IKR) in the cell plasma membrane. These channels open when  $V_{\rm m}$  is equal to or more negative than the potassium equilibrium potential and close at more positive values. Noble published the first successful computer-assisted, heuristic model description of the contribution of the IKR conduc-

tance to the membrane potential of cardiac cells, and its influence leading to the depolarisation of the membrane potential  $V_{\rm m}$  in low potassium media [1,2]. Since then, a number of similar more accurate models have been produced to explain or describe the heart action potential and its propagation. Meanwhile, inward rectifying currents and channels have been found in many other cell systems. In a number of publications the direct current-voltage relationship has been measured [3-6]. Other publications report the

observation of depolarisation on lowering the medium potassium concentration [7-11], the existence of two stable values of  $V_{\rm m}$  in the same preparation [8,12], the observation of hyperpolarisation from a low to high  $V_{\rm m}$  (negative inside) on stimulation of the cell [13], or the increase in Na/K pump site density on systemic reduction of  $K_{\rm o}$  in the culturing medium [14]. Furthermore, there is a continuing discussion why some types of skeletal muscle fibres hyperpolarise and others do not [7-10] when  $K_0$  is reduced. If the properties of the IKR, indeed, form the main origin of the dichotomy in  $V_m$  as a function of  $K_0$ , one must conclude from these papers, that, leaving apart experimental differences, this potassium channel does not always exhibit exactly the same behaviour in all cells [15]. At the same time it is found so frequently in so many cell types, that it is most unlikely that this phenomenon depends critically on detailed molecular structures of the channels. So, the question arises: "Is inward rectification of this potassium channel alone sufficient to explain the observed dichotomy in  $V_m$ ?"

As described by Jack et al. [16], any conductive cation system exhibits inward rectification as soon as its "controlling step" determining its resistance is located more at the intracellular side of the membrane than at the extracellular side. So, a broad range of molecular channel structures may exist to explain inward rectification. It has been described in a great variety of cells such as cardiac cells [1,4,8,12,17,18], starfish egg cells [19], skeletal muscle fibres [3,7,9,10,11,20-23], osteoclasts [5], neurones and glial cells [6,13,24], epithelial cells [25], erythrocytes [26] and macrophage cells [27]. This might also be why frequently this type of membrane conductance has been given different names such as " $I_{K,leak}$ " in textbooks, "I<sub>K1</sub>" in cardiac cells [16,18], "anomalous rectifier: AR" [10,20], inward rectifier K+ channel, IRK [27], and more complex according to molecular description [15]. For reasons of generality, the name "inward K+-rectifier IKR" (and the current through it  $I_{KR}$ ) is preferred here.

Given the particularly strong dependence of the IKR on  $V_{\rm m}$  and  $K_{\rm o}$  it is impossible to describe the functional dependence of  $V_{\rm m}$  on the extracellular ion concentrations in an explicit

form. All variables are mutually dependent, prohibiting an identification of what are the dependent and independent variables in the, now classic, equations given by Goldman, Hodgkin and Katz ([28] Eq. (4.13)) and Mullins and Noda ([28] Eq. (4.16)). Not withstanding this difficulty, these equations still apply to a system in steady state. Moreover the Na/K pump that maintains the disequilibrium in the concentrations of sodium and potassium also depends kinetically on  $K_0$  [28].

Whereas recent detailed models to explain cardiac rhythmicity [29–32] include IKR, they are frequently so heavily connected with experimentally obtained results, that the generic question about the relationship between ion concentrations, membrane potentials and ion conductances is overshadowed by the details. Likewise, they do not deal with conditions where  $K_0$  is reduced. Consequently, investigators in other fields hesitate to exploit these models, when they observe in their cell systems similar peculiarities as mentioned above but do not dispose over such a wealth of experimental data.

The present contribution investigates what characteristic properties of IKR make  $V_{\rm m}$  depolarise on lowering  $K_{\rm o}$ . As a framework for formulating the influence of IKR on cellular homeostasis, the system is broken down into easy identifiable elements, which can simply be exchanged and mathematically reformulated. In its turn, the model may indicate what properties and elements will contribute to the depolarization of  $V_{\rm m}$ . This study investigated only those properties of IKR as function of time, that do not exhibit short lasting transient behaviour.

Voltage clamp and patch clamp studies have revealed several properties of IKR and have led to a number of detailed descriptions concerning its molecular structure and functioning [3,15, 23,27,33–38]. The model uses a rather simple analytical relationship resembling the analysis by Hagiwara and Takahashi [19]. Furthermore, for sake of simplicity, the model treats the cells as non-polar and without unstirred layers adjacent to the membrane.

As most "macroscopical" cellular membrane conductances the total potassium conductance is composed of several kinds of potassium conductances exhibiting each their own kinetics and sensitivities. These kinetics can be traced down to the open/close probabilities of the individual ion channels in the membrane that respond according to their own sensitivities to transmembrane potential (voltage sensitive channels) or other stimuli. The channels that underlie IKR are a clear example of such voltage sensitive channels. The moiety of the channel that is endowed with this ability is thought to possess a so-called "gating charge" [33,39,40], that moves in the electrical field created by the membrane potential. By statistical summation of all open and closed channel possibilities, using the Boltzmann distribution, one obtains the macroscopical conductance. Usually the characteristic elements for the macroscopical current are formulated by the membrane potential when the conductance is half-maximal (here:  $V_h$  (mV)) and the slope of change (here:  $V_s$ (mV)). The value of the gating-charge is inversely related to the slope [39]. Such approach has turned out to be very fruitful and can be applied to most channels known. However, the partition function may be much more complex if there are more choices than only "open" and "close". The model does not introduce this type of complexity, because experimental data do still not definitely make clear how this complex behaviour should be described. Moreover, the singularity in the dependence of  $V_m$  on  $K_0$  occurs so frequently, that precise details of channel kinetics can not be the origin of this singularity. Whereas at  $V_{\text{rest}}$  IKR dominates, the closure of IKR will produce a new steady state  $V_{\rm m}$  and make other ion currents (including other types of potassium currents, like the calcium-activated potassium current  $I_{KCa}$ ) to become more apparent. This introduction makes clear that the rather colloquial expression "IKR closes", used frequently, should be understood as "the "closed"-probability in the ensemble average of all inward rectifying potassium channels of the cell becomes larger". The expression "switchoff" means that "the closed probability becomes heavily predominant".

This contribution breaks down into five parts. Section 2 presents one new experimental graph, showing the basic phenomenon of the dichotomy in  $V_{rest}$  frequently observed experimentally by us and others [1.7-10]. Section 3 introduces the model in generic terms based on the principles of compartmental analysis. The elementary relationships of this generic model are open for redefinition, when such relationship has experimentally been determined and formulated. Section 4 demonstrates the implementation of the model for mouse skeletal muscle fibre with the relationships specifically applicable to that situation. Section 5 analyzes the sensitivity of the model for the choice of the individual parameters. In this section it is also demonstrated that the two versions of the model introduced in section 3.2, one for long-term flux measurements and one for shortterm voltage-clamp measurements, are complementary. In section 6 the results are discussed and a number of possible refinements, that might be proposed on grounds of data dealing with elements of the total model. It also provides a prediction, that has turned out to be likely on grounds of data being published after the simulations were made.

# 2. Experimentally measured responses of $V_{\text{rest}}$ to different $K_0$

To make the basic observation understandable, in Fig. 1 the values of  $V_{\rm rest}$ , measured with intracellular microelectrodes in lumbrical muscle fibres of mice, are plotted at different  $K_{\rm o}$  values on a semi-logarithmic scale. All experimental procedures are the same as described previously [10]. Down to 2 mM,  $V_{\rm rest}$  decreases steadily with  $K_{\rm o}$  following a slope of 55 mV/decade.

At  $K_{\rm o}$  values between 1.0 and 2 mM,  $V_{\rm rest}$  may be depolarized (to -52 mV) or hyperpolarized (about -94 mV), but below  $K_{\rm o}=1.0$  mM only depolarized  $V_{\rm rest}$  values were found. Similar relationships have been found in other cells [1,7-10,24] and this effect will be called here "switch-off". Such switch-off would not be observed if all membrane permeabilities remained constant [16,28,39,41]. Deterioration of cells is not the reason for this dichotomy (or singularity) of the dependence of  $V_{\rm rest}$  upon  $K_{\rm o}$  [10]. Sometimes  $V_{\rm rest}$  starts to hyperpolarise on applying low  $K_{\rm o}$ ,

but depolarizes later without any apparent reason. After returning to the control solution, this behaviour can be re-evoked in the same cell. Additionally, blocking the Na/K pump with ouabain can also depolarize  $V_{\rm rest}$  to a value of about -52 mV, along with a strong reduction of the relative potassium selectivity as has also been reported [10,11,42].

### 3. Methods and generic description of the model

#### 3.1. Used program and computer

The simulation package used (STELLA<sup>TM</sup>; version 2.10 (High Performance Systems, Lyme, New Hampshire 03768, USA) running on Apple Macintosh) is based upon a compartmental analysis approach to integrate step-wise differential equations. The routinely provided fourth-order Runge-Kutta method is used in the model. The graphic presentations of the simulation results in this paper were made with a more sophisticated program; in figures showing data as a function of time, the calculated points are concealed under the fitting curves. The simulation program does not contain more advanced integration algorithms to vary dynamically the scaling of the integration steps [30,31]. Because conductance

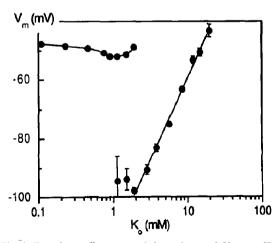


Fig. 1. Experimentally measured dependence of  $V_{\rm rest}$  on  $K_{\rm o}$  in the lumbrical muscle fibres of mice. The slope of the line fitting  $V_{\rm m}$  values for  $K_{\rm o} \geqslant 2$  mM is 55 mV/decade. In the range between 1 and 2 mM  $K_{\rm o}$  depolarization as well as hyperpolarization was obtained, whereas below 1 mM only depolarized  $V_{\rm rest}$  was found. Bars represent  $\pm$  standard error of the mean (n=6).

and voltage changes differ in time by several orders of magnitude ([28] p. 13) from ion concentration changes, simulations that incorporate both might be either time consuming or to suffer from numerical instabilities. To overcome this problem, two complementary versions of the model were used. One version was for the voltage clamp simulations (assuming no change in cell concentrations due to simulation protocol). The other was for the ion-substitution experiments, taking advantage of the fact that in these experiments always electroneutral replacements of one salt by another in the medium were made.

## 3.2. Structure of the model

The general scheme used in the model is presented in Fig. 2.

The model is built with three types of equations: conservation equations, force-flux equations and one equation describing the steady state kinetics of the Na/K pump. The conservation equations are all derived from the general equation that integrates the influx of charge  $(I = \Sigma I_j, w)$  with the ion species  $j = Na^+, K^+, Cl^-)$  onto the membrane capacitance  $(C = C_m S)$  with  $C_m$  the specific membrane capacitance and S the surface of the cell) changing the membrane potential  $(V_m)$ . Likewise it integrates the efflux of molecules  $(J_j, w)$  with charge  $z_j$ ; F is the constant of Faraday) onto the volume of the cell (V) changing their concentration  $(c_i)$ :

$$V_{\mathbf{m}}(t) - V_{\mathbf{m}}(0)$$

$$= -\int_{0}^{t} (I/C) \, \mathrm{d}t'$$

$$= -\int_{0}^{t} \left( \sum_{j} I_{j} / (C_{\mathbf{m}}S) \right) \, \mathrm{d}t'$$

$$= -\int_{0}^{t} \left( \sum_{j} Fz_{j} J_{j} \right) / (C_{\mathbf{m}}S) \, \mathrm{d}t'$$

$$= -\int_{0}^{t} \left( \sum_{j} z_{j} \, \mathrm{d}c_{j}(t') / \, \mathrm{d}t' \right) (FV/C_{\mathbf{m}}S) \, \mathrm{d}t'$$

$$(1)$$

The general form of all force-flux equations for passive diffusion is still formulated with use of the constant-field flux equation [16,28,41]:

$$J_j = U(c_{ji} e^U - c_{jo}) P_j / (e^U - 1),$$
 (2)

where  $U = z_j F V_m / RT = z_j V_m / 0.025$ ,  $P_j$  is the membrane permeability for j; subscript o and i stand for outside and inside the cell, respectively.

The formulation of  $P_j$  incorporates all voltage or concentration dependencies of permeabilities (especially those of IKR). The model does not assume active transport or particular voltage-dependent conductance changes for chloride and applies Eq. (2) directly to this ion with a constant value for  $P_{\text{Cl}}$ . For sodium and potassium, active transport by the Na/K pump adds, with the proper sign,  $3J_{\text{pump}}$  to  $J_{\text{Na}}$  and  $2J_{\text{pump}}$  to  $J_{\text{K}}$ . Concentration dependence on Na<sub>i</sub> and  $K_{\text{o}}$  of  $J_{\text{pump}}$  is implemented with the equation frequently used in simulations [31,32],

$$J_{\text{pump}} = J_{\text{p,max}} (1 + K_{\text{mK}} / K_{\text{o}})^{-2} \times (1 + K_{\text{mNa}} / \text{Na}_{\text{i}})^{-3},$$
 (3)

thus introducing three parameters for the model: the maximal pump-activity:  $J_{\rm p,max}$  and the affinity constants for Na<sub>i</sub>:  $K_{\rm mNa}$  and for  $K_{\rm o}$ :  $K_{\rm mK}$ . As a first approximation, the dependence of  $J_{\rm pump}$  on  $V_{\rm m}$  [28,43] is not installed and  $P_{\rm Na}$  is a constant.

The relation between changes in  $V_{\rm m}$  and  $c_j$  due to membrane fluxes of ion j is

$$\left(\frac{\mathrm{d}V_{\mathrm{m}}}{\mathrm{d}t}\right)_{i} = \frac{\phi_{\mathrm{d}}FV}{C_{\mathrm{m}}S} \frac{\mathrm{d}c_{i}}{\mathrm{d}t} = q \frac{\mathrm{d}c_{j}}{\mathrm{d}t}.$$
 (4)

Included in q is a factor  $\phi_d$  that allows for an adaptation for inaccessible space in the cell. It was taken as being 1 in the voltage clamp version and as 0.9 in the ion flux version rather arbitrary until a better determination is available. Normally q (dimension V/M) in this equation, relating the electrical changes to the concentration changes of individual ionic fluxes, is so large, that

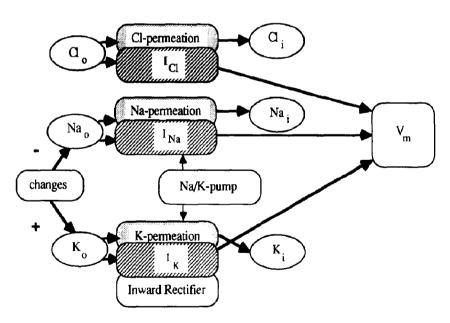


Fig. 2. Schematic presentation of the model, showing the changes, fluxes and integrations in the model. "Changes" represents the exchanging of  $K^+$  by  $Na^+$  or vice versa. To mimic the fact that experimentally such changes take some time, the step function is smoothed with a time constant of about 8 s (except in Fig. 5b, where this constant is varied).  $Cl_o$ ,  $Na_o$ ,  $K_o$ ,  $Cl_i$ ,  $Na_i$  and  $K_i$  represent the concentrations of chloride, sodium or potassium outside or inside the cell.  $Cl_i$ ,  $Cl_i$  and  $Cl_i$  and  $Cl_i$  and  $Cl_i$  fluxes represented here as arrows); the concomitant electrical currents are  $Cl_i$  fluxes and  $Cl_i$  and  $Cl_i$  and  $Cl_i$  and  $Cl_i$  and  $Cl_i$  and  $Cl_i$  are effected by integration of the electrical current (Eqs. (1) and (4)). The contributions of the  $Cl_i$  and  $Cl_i$  and  $Cl_i$  are derived from Eq. (3). The contribution of IKR to potassium permeation is explicitly shown and is implemented by Eq. (7). In the voltage clamp version the changes were kept zero, but a command signal  $Cl_i$  command (not shown in this figure) is imposed upon  $Cl_i$  by a current:  $Cl_i$  command = gain( $Cl_i$  and  $Cl_i$  in the voltage clamp version the changes were kept zero, but a command signal  $Cl_i$  command (not shown in this figure) is imposed upon  $Cl_i$  by a current:  $Cl_i$  in this version the real conversion between fluxes and currents  $Cl_i$  is used and the total actual time could be reduced to seconds.

small changes in concentration are associated with a large change in  $V_{\rm m}$ . This poses stringent limitations to the stability of the numerical integration: time steps for integration in electrical terms should be essentially much shorter than the time steps one would like to use in the simulation of flux experiments. As all parameters are electrically heavily fed back by Eqs. (1) and (2) (see also section 1), one can assume that the fast electrical processes are always in steady state on the time scale of the slower diffusional processes [28]. Therefore, a reduction of the value for q in the flux simulations will not considerably change the outcome. This was verified (see section 5.1) and led to the installation of two separate versions of the model: one mainly for the description of ion substitution experiments (taking  $a \approx 1\%$  of the proper value) and the other for the simulation of voltage clamp experiments (with q as the proper value).

The model uses values for the relative membrane permeabilities for K<sup>+</sup>, Na<sup>+</sup> and Cl<sup>-</sup> obtained from the literature. Trial-runs of the model provided parameter values to fit to other data available from the literature, such as the intracellular concentrations (see Fig. 3). By this method one needs to specify only one absolute permeability value, which can be deduced from the measurement of the permeability or of the conductance of the membrane for one ion that is described by the model (see section 4.1.).

An expression for the potassium permeability:  $P_{\rm K}$ , is needed, that contains the relation of the IKR to its most important parameters: its conductance when fully opened, the steady state open/close partition function of the channels, and a residual permeability ( $P_{\rm o}$ ) when IKR is fully closed.

Conductances can be described as chord conductance [16,41] defined as  $g_{K(\text{chord})} = I_K/(V_m - E_K)$  or as slope conductance  $g_{K(\text{slope})} = (\partial I_K/\partial V_m)$ . For  $V_m$  near  $E_K$  (i.e.  $V_m - E_K \approx 0$ ) the difference vanishes ( $E_K$  being the equilibrium potential for  $K^+$ ),  $g_{K(\text{chord})} \approx g_{K(\text{slope})}$ .  $P_K$  is related to  $g_K$  by the following approximation [16,41]:

$$P_{K} = g_{K}RT/(zF)^{2}c_{K.membrane}.$$
 (5)

The K<sup>+</sup> concentration in the membrane,

 $c_{\rm K,membrane}$ , can be approximated by the logarithmic average between the concentration on either side: in the cell  $(c_{\rm K,i})$  and outside  $(c_{\rm K,0})$  [41]:

$$c_{K,\text{membrane}} = (c_{K,o} - c_{K,i}) / \ln(c_{K,o} / c_{K,i}).$$
 (6)

The dependence of  $P_K$  on  $K_o$  and  $V_m$  is defined similar to the expression proposed by Hagiwara and Takahashi [19] for  $g_K$ ,

$$P_{\rm K} = P_{\rm o} + (\overline{P}_{\rm K}/\sqrt{K_{\rm o}})[1 + \exp(V_{\rm m} - V_{\rm h})/V_{\rm s}]^{-1}.$$
 (7)

 $P_{\rm o}$  is the residual potassium permeability needed in the model to account for the experimentally observed fact that closure of the IKR let  $V_{\rm m}$  depolarize to about -50 mV instead of to the equilibrium potential of Na<sup>+</sup>,  $E_{\rm Na} \approx +60$  mV (see Fig. 1). The appropriate value was  $P_{\rm o} \approx 4P_{\rm Na}$ . As mentioned in section 1, this residual permeability may consist of a number of conductances, which may be open already or become opened because of the depolarization of  $V_{\rm m}$  (like  $I_{\rm KCa}$ ).

The function in the denominator of the righthand side of Eq. (7),  $(1 + \exp(V_m - V_h)/V_s]$ , is the steady state open/close partition function of the channels according to the Boltzmann distribution. In principle one can incorporate in this function all steady state kinetic peculiarities of the IKR, such as its gating mechanism [3,15,17,23,33,34,36], its dependence on  $K_0$ [3,4,7-11,19,22] and its ion-selectivity [18,35,37]. In Eq. (7)  $V_h$  is the value where this term is half-maximal. The parameter  $V_s$ , frequently called the slope parameter, defines the sensitivity of the partition function for the energy differences (here  $V_{\rm m} - V_{\rm h}$ ) and is related to the gating charge [33,39]. Hagiwara and Takahashi [19] showed that over a wide range of  $K_0$  values  $V_h$  differed by a constant value from  $E_{K}$ : so,  $V_{h}$  depends on  $K_{o}$ . In section 4 the values  $V_h$  and  $V_s$  will be adapted to values in skeletal muscle fibres. In the sensitivity analysis (section 5) the influence of the choice of  $V_{\rm h}$  and  $V_{\rm s}$  will be considered. More complex kinetics will be discussed in section 6.

The value  $K_o$  enters the expression of  $P_K$  explicitly because of the factor  $\sqrt{K_o}$  and implicitly because it also affects  $V_h$ , that is dependent on  $E_K$ . The first term of the description of the

permeability of IKR,  $\bar{P}_{\rm K}/\sqrt{K_{\rm o}}$ , represents the maximal permeability of IKR when  $(V_{\rm m}-V_{\rm h})/V_{\rm s} \ll 0$  (and the partition function  $\approx 1$ ). It has been observed frequently that  $\bar{g}_{\rm K}$  (the maximal conductance of IKR) varies proportionally to the square root of  $K_{\rm o}$  [4,18,19,36,39]. In view of Eqs. (5) and (6), the most appropriate way to include this fact is to install  $\sqrt{K_{\rm o}}$  in the denominator of this component.  $\bar{P}_{\rm K}$  allows the scaling of the total model in such a way that at one value of  $K_{\rm o}$  this component fits with the experimentally observed value.

The process of voltage clamping was simulated by including a voltage command mode, injecting electrical current into the cell to keep  $V_{\rm m} = V_{\rm command}$  within 1%. The experimental uncertainty in this situation as to which ions carry the injected current, and the short duration of the current injection, made us assume that the current injection does not affect the intracellular concentrations.

The model simulates delays, normally induced by diffusion or by mixing processes in many experimental studies, by smoothing the step-function that simulates the changes of the medium composition (see Fig. 2 and section 4.3).

In the model no explicit boundary conditions were introduced ensuring constant osmotic pressure in the cell and only inspection of the total change of intracellular K<sup>+</sup>, Na<sup>+</sup> and Cl<sup>-</sup> ions was carried out. The most important reasons for not imposing stringent osmotic limitations are: (1) most experimental data do not provide information about this aspect; (2) it is likely that cells swell or shrink when the concentrations vary; and (3) volume- and osmo-regulation by means of ion-cotransporters are very likely to occur in reality [44], but functional relationships to be inserted in the model are missing as yet.

#### 4. The model applied to skeletal muscle fibres

### 4.1. Installation of the model

In this section the model is applied to a simulation of the resting potential  $V_{\text{rest}}$  in mouse skeletal muscle fibres using data from the literature and our own observations. The complex defi-

nition of  $V_h$  introduced here is applicable to this section but is no prerequisite to demonstrate that lowering  $K_0$  leads to a depolarization (see 5.2.2). The fibre was described as a cylinder in Eq. (4); using the following values:  $r = 17.5 \mu m$  [45] and  $C_{\rm m} = 8 \ \mu \text{F/cm}^2$  [46]. The relation between the electrical and concentration changes is q = $\phi_d FV/C_m S(=Fr/2C_m) = 10552$  V/M, with  $\phi_d$ = 1. From ref. [21] the ratios  $\alpha_{\rm m} = P_{\rm Na}/P_{\rm K} = 0.01$ and  $P_{\rm Cl}/P_{\rm K} = 3$  were taken to apply to the control situation (here  $K_0 = 5.7$  mM). The value of  $g_{K}$  at 100 mM  $K_{o}$ , as provided by Standen and Stanfield [3] (3.4 mS cm<sup>-2</sup>), was used to scale  $P_{K}$ according to the experimental value of  $7.6 \times 10^{-6}$ cm/s using Eqs. (5) and (6) and taking  $K_i = 175$ mM (see Fig. 4a). Their  $g_K$  values in media with different potassium concentrations were fitted with an equation comparable to Eq. (7) and provided the dependence of  $V_h$  on  $K_o$ . For  $K_o > 10$ mM the values for  $V_h$  differed by a constant value from  $E_{K}$ , comparable with data obtained by others [19,31,32]:  $V_h - E_K = \Delta V_{h,K-high} = -24$ mV. For  $K_0$  is 2.5 and 10 mM  $V_h = -72$  mV [3], and with  $K_0 = 2.5$  mM  $V_h > E_K$ . Voltage clamp data in the literature do not provide much information about  $V_h$  at values of  $K_o$  lower then 2.5 mM, because of the difficulty to measure the reduced currents through IKR when  $K_{\alpha}$  is low. The assumption that  $V_h$  remains constant for all  $K_0 \le 10$  mM would imply that the partition function of Eq. (7) becomes a constant provided that  $V_{\rm m}$  also levels off. This would imply that  $P_{\rm K}$ becomes approximately a constant and  $V_m$  behaves as described by the classic Goldman-Hodgkin-Katz equation [28,41]. A hyperpolarized  $V_{\rm rest}$  means that  $P_{\rm K}$  remains sufficiently large or, alternatively formulated,  $V_{\rm m} - V_{\rm h}/V_{\rm s}$  sufficiently small (Eq. (7) and section 5.2.2). The model assumes that from about 10 mM downwards  $V_h$  begins to differ again from  $E_K$ , but now positively (with as maximal constant value  $\Delta V_{\rm b,K} = \text{low} = +30 \text{ mV}$ ). This rather complex relationship between  $V_h$  and  $E_K$  was described again with a Boltzmann-like function for the dependence of  $V_h$  on  $K_o$  (see curve " $V_h$ " in Fig. 4c):

$$V_{\rm h} = E_{\rm K} + \Delta V_{\rm h,K-low} - \frac{2(\Delta V_{\rm h,K-low} - \Delta V_{\rm h,K-high})}{1 + \exp(K_{\rm co}/K_{\rm o})}. \tag{8}$$

The value  $K_{\infty}$  represents the cross-over concentration of  $K_{\infty}$ , where  $V_{\rm h} \approx E_{\rm K}$  (see Fig. 4:  $K_{\infty} \approx 13$  mM).

As slope parameter in Eq. (7) was chosen  $V_s = 9$  mV, which is equivalent to a "gating charge" of 2.8 [28,33,39].

The Na/K pump was characterized by taking  $K_{\rm mK}=1$  mM [47] and adjusting  $J_{\rm p,max}$  and  $K_{\rm mNa}$  to obtain an integrated set of values for  $c_{\rm j}$  and  $V_{\rm rest}$ , comparable to experimentally determined values, at  $K_{\rm o}=5.7$  mM [10,48]: 130 mM <  $K_{\rm i}$  < 150 mM; 7 mM < Na<sub>i</sub> < 20 mM; 5 mM < Cl<sub>i</sub> < 7 mM;  $-77 < V_{\rm rest} < -74$  mV (the value of  $V_{\rm rest}$ 

determines directly  $Cl_i$  by means of the Nernst equation). The values  $J_{p,max} = 50 \text{ pmol/cm}^2 \text{ s}$  and  $K_{mNa} = 7 \text{ mM}$  were chosen (see Fig. 3) and used routinely further for all simulations. Fig. 3d demonstrates two important aspects:  $J_{pump}$  is, under all conditions specified, never more than 30% of  $J_{p,max}$  and under control conditions the choice of  $K_{m,Na}$  is not very important anymore.

## 4.2. Dependencies on K

The changes induced by substituting  $K_o$  for Na<sub>o</sub>, or vice versa, in the steady state values of

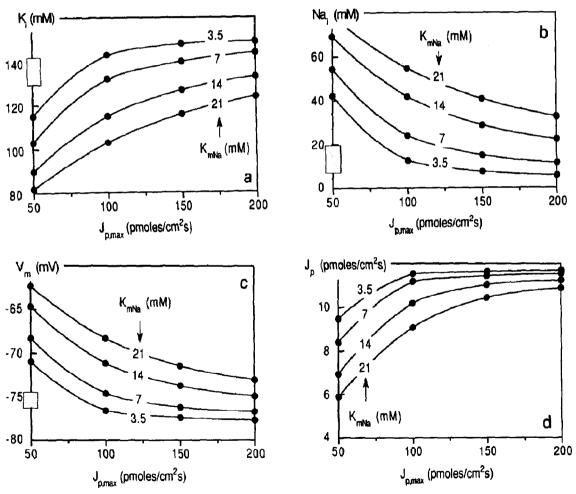


Fig. 3. Dependence on  $J_{p,max}$  (ranging 50 to 200 pmol Na/cm<sup>2</sup> s) and  $K_{mNa}$  (3.5, 7, 14 and 21 mM) of: (a)  $K_i$  (mM); (b) Na<sub>i</sub> (mM); (c)  $V_m$  (mV); (d)  $J_p$  (pmol/cm<sup>2</sup> s). The experimental values for  $K_i$ , Na<sub>i</sub> and  $V_m$  are indicated by a bar on the vertical axis in each panel; the choice  $J_{p,max} = 150$  pmol Na/cm<sup>2</sup> s and  $K_{mNa} = 7$  mM makes the simulated values fall in the indicated range. All values presented were obtained at the end of 1800 s simulations. Values for  $Cl_i$  are not shown: they can be found using the Nernst equation and fall in the range as specified.

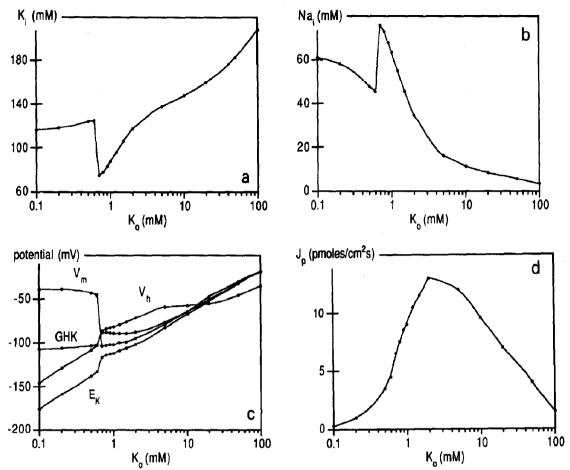


Fig. 4. Dependence of cellular parameters, calculated 2900 s actual time after the change in  $K_o$  from 5.7 mM, on the  $K_o$  value specified on the horizontal axis: (a)  $K_i$ ; (b)  $Na_i$ ; (c)  $V_m$ ,  $V_h$ ,  $E_K$ , GHK and (d)  $J_p$ . Curves were obtained by smoothed interpolation; for  $J_p$  two lines were used: one from 0.1 to 2 mM  $K_o$  and one for  $K_o > 2$  mM. The curves in panels a, b and c show deviations near the switch-off value,  $K_o = 1$  mM, because a fully steady state in this critical range is not attained after 50 min; to attain such values unrealistic long times are required. When  $K_o$  decreases,  $K_i$  decreases as well, until IKR closes. Then  $K_i$  increases again. The response of  $Na_i$  mirrors that of  $K_i$ . Increased  $Na_i$  can compensate for the reduced stimulation of the Na/K pump by  $K_o$ .  $Cl_i$  closely follows  $V_m$  (see panel c). In panel c GHK represents the membrane potential calculated from  $K_i$  and  $Na_i$  using the GHK equation with a constant  $P_K/P_{Na}$  ratio (= 100). The curve  $V_h$  closely follows Eq. (8); at  $K_o \approx 13$  mV  $V_h = E_K$ .

 $K_i$ , Na<sub>i</sub> and  $J_{\text{pump}}$  are displayed in Fig. 4 as functions of  $K_o$ . Several aspects can be identified.  $J_{\text{pump}}$  has its maximal value in the range between 5 and 2 mM and starts to decline at about 2 mM  $K_o$  as can be seen in Fig. 4d, but also in Figs. 4a and b where at this concentration  $K_i$  and Na<sub>i</sub> begin to deviate from the extrapolation of curves obtained at higher  $K_o$  values.

When  $K_0$  is lowered below the critical value of 0.7 mM the closure of IKR leads to an increase of  $K_i$ , though the electrochemical gradient

increases due to both its electrical and chemical component. The consequence of the reduction in  $J_{\text{pump}}$  is also visible in the curves for  $V_{\text{m}}$ , and GHK in Fig. 4c and particularly in  $E_{\text{K}}$  demonstrating a hump in this part of the curve. GHK is calculated using the Goldman-Hodgkin-Katz voltage equation [39,41] with  $\alpha_{\text{m}} = P_{\text{Na}}/P_{\text{K}} = 0.01$  and the new steady state values of  $K_{\text{i}}$ , Na<sub>i</sub> and Cl<sub>i</sub>; all as a function of  $K_{\text{o}}$ . In Fig. 4c the value of  $V_{\text{h}}$  is also shown. The sum of the intracellular monovalent ion concentrations remains compara-

ble to the initial value when  $K_0$  is between 0.6 and 10 mM.

Because  $V_h$  decreases with decreasing  $K_o$ , according to Eq. (8) a switch is always observed. The exact value of  $K_o$ , where this switch occurs, can be influenced by the properties of IKR (for instance, its dependence on  $V_s$  and  $V_h$ ) and those of the Na/K pump (for instance, its dependence on  $K_{mK}$  and  $J_{p,max}$ ). Instead of scaling further the unknown parameters to fit as good as possible our own experimental results, in section 5 the influence of the critical parameters on the behaviour of the simulated cell is analyzed. In addition, experimental data [18,49,50] suggest that insertion of a dependence of IKR on intracellular ion concentrations might be more appropriate (see section 6).

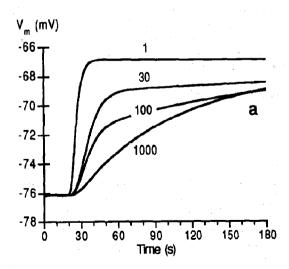
## 4.3. Influence of chloride on $V_m$ responses to cation substitutions

In the model, chloride distributes passively and it should not influence the value  $V_{\rm rest}$ . However, the chloride permeability does influence the velocity with which  $V_{\rm m}$  can respond to ion changes in the medium. This is illustrated in Fig. 5a, where increasing  $P_{\rm Cl}$  leads to a slowing down of

the speed of depolarization induced by a sudden increase of  $K_0$ . The results compare qualitatively well with observations made by Hodgkin and Horowicz ([21] Fig. 9). Thus, the process of redistribution of intracellular chloride slows down the readjustment of  $V_{\rm m}$  to a value near  $E_{\rm K}$ . If  $K_{\rm o}$ changes fast compared to the response time of  $V_{\rm m}$ , the difference between  $V_{\rm m}$  and  $V_{\rm h}$ , that follows  $E_{\rm K}$  (Eq. (8)), immediately exceeds the critical value in Eq. (7) and  $P_{\rm K}$  drops to  $P_{\rm o}$  leading to a direct cell depolarisation ("Fast" in Fig. 5b). When  $K_0$  changes more gradually (two curves called "Slow" in Fig. 5b), it may take a considerable time (> 2 h) to find out that the hyperpolarized  $V_{\rm m}$  is not a stable value. Then  $V_{\rm rest}$  depolarizes due to loss of intracellular potassium because IKR remains open (the total  $P_{K}$  being larger than in the control situation as it depends on  $1/\sqrt{K_0}$ : cf. Eq. 7). Experiments [10] where  $K_0$ was first reduced to 1.9 mM, and after 30 min to 0.76 mM, did not show such accommodation.

### 4.4. Voltage clamp experiments

Because the description of the properties of IKR is based on a number of published voltage-clamp experiments, it was worthwhile to see to



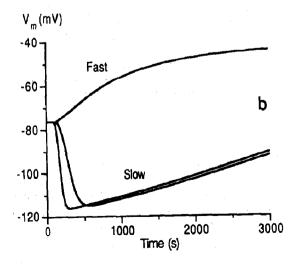


Fig. 5. The influence of Cl<sup>-</sup>. (a)  $P_{\rm Cl}$  (values in the curves are  $P_{\rm Cl}/P_{\rm Na}$ ) delays the depolarization of  $V_{\rm m}$  in response to increasing  $K_{\rm o}$  (at t=20) from 5.7 to 8.55 mM. (b) When  $P_{\rm Cl}$  is large (300  $P_{\rm Na}$ ), lowering  $K_{\rm o}$  quickly to 0.7 mM can lead to a depolarization of  $V_{\rm m}$  ("Fast": smoothing the step function, that simulates the change in ion composition, with a time constant of 4 s) whereas lowering  $K_{\rm o}$  slowly leads to an hyperpolarization ("Slow": two curves with smoothing time constants of 40 and 140 s, respectively). "Smoothing" is a routinely provided function in the program.

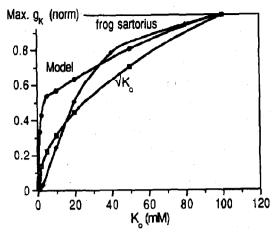


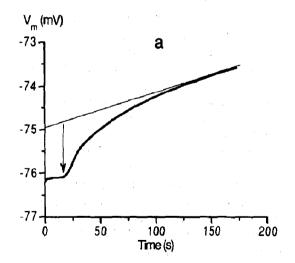
Fig. 6. Voltage clamp results. The dependence of  $\bar{g}_K$  of  $K_0$  is compared with  $\sqrt{K_0}$  and the experimental data for sartorius muscle [3]. All values are normalized to 1 for  $K_0 = 100$  mM.

what extent the voltage clamp version of the model can simulate the findings of these studies. The model simulated the experiments of Standen and Stanfield [3] at  $Cl_0 = 0$  by taking  $P_{Cl} = P_{Na}$ . In the voltage clamp version the values obtained for  $I_{KR}$  were divided by  $V_m - E_K$  to produce  $g_K$  and not calculated from  $P_K$  according to Eq. (7). The data are presented in Fig. 6a together with

the function  $1/\sqrt{K_o}$  and the data from ref. [3]. All curves are normalised to make their values 1 for  $K_0 = 100$  mM; the value of  $\bar{g}$ K found with  $K_0 = 100$  mM is about two times larger then the actual value from [3]. The dependence of the simulated  $\bar{g}_{K}$  on  $K_{0}$  compares better with the square root curve than with the data of the experiments. Once IKR is fully opened at hyperpolarised  $V_{\rm m}$ , the normal Goldman-Hodgkin-Katz equation can describe  $\bar{g}_{K}$  as function of  $V_{m}$ [16,28,39,41]. In Fig. 6 the voltage clamp responses were calculated with intracellular concentrations equal to the values, when  $K_0 = 5.7$ mM. Calculations with the actual steady state values as given in Fig. 4 showed only 10 to 15% deviations from the values in Fig. 6.

### 4.5. The effect of ouabain

Experimentally, ouabain induces a greater depolarization in skeletal muscle fibres [10,42] than the 1 mV predicted by the usual straightforward theoretical analysis [28]. The instantaneous effect of ouabain was simulated by turning  $J_{\rm p,max}$  to zero and Fig. 7a shows that the model calculation gives an initial depolarization of about 1 mV. In



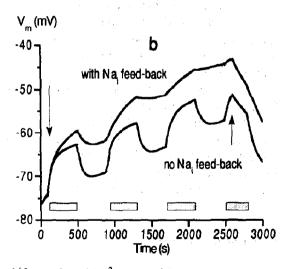


Fig. 7. The influence of ouabain. (a) When the Na/K pump is stopped  $(J_{\rm p,max}=0~{\rm pmol/cm^2~s}$  at t=20) the initial depolarization, specified as the difference (arrow) between  $V_{\rm rest}$  ( $t \le 20$ ) and the value at t=20 of the line, tangent to the curve at 180 s, amounts about 1 mV. (b) The "long-term" response of  $V_{\rm m}$  to ouabain (applied between t=100 and 2600: see arrows). Periodical changes in  $K_0$  from 5.7 to 8.55 mM (bars) induce variations in  $V_{\rm m}$  that give an impression of the selectivity ratio  $P_{\rm K}/P_{\rm Na}$ : this ratio being correlated to the size of the response. Two conditions were compared: (1) "no Na<sub>i</sub> feed back": when  $P_{\rm K}$  is described as in this paper (Eq. (7)) and the ratio does not show a decrease, and (2) "with Na<sub>i</sub> feed back": when  $P_{\rm K}$  is multiplied by a factor (Na<sub>i,control</sub>/Na<sub>i,exp</sub>) thus simulating the condition that  $P_{\rm K}$  is inhibited by Na<sub>i</sub>.

Fig. 7b the simulation of  $V_{\rm m}$  during longer exposures to ouabain is shown. The response of  $V_m$  on varying  $K_0$  between 5.7 and 8.55 mM (150%) gives an impression of the selectivity ratio  $\alpha_m$  =  $P_{\rm Na}/P_{\rm K}$ . Experimentally, these responses of  $V_{\rm m}$ to such variations in K<sub>o</sub> decreased continuously after ouabain application (0.1 mM) indicating that this ratio increased [10]. In the simulation curve indicated by "no Na;-feedback" the responses remained constant, indicating that according to the model  $P_{Na}/P_{K}$  is not increased by inhibition of the pump alone. Here the depolarization is due to a drop in  $E_K$ , and by the definitions of  $V_h$ (Eq. (8)) and  $P_K$  (Eq. (7)) there is no reason for a closure of IKR because  $V_h$  and  $V_m$  follow  $E_K$ closely, thus keeping the partition function small. The second trace in Fig. 7b shows that the decrease in K<sup>+</sup>-selectivity can be mimicked, in this case by installing a reduction of  $P_K$  by increasing Na: [18,50]. It compares rather well with the experimental observation ([10], Fig. 7). Another alternative, increase of  $P_{Na}$  by increasing  $Na_i$ , as suggested by others [42] is also possible and can be simulated as well. Kawata and Hatae [51] demonstrated that addition of Ba2+ to the medium perfusing frog skeletal muscle leads to an initial increase of the membrane resistance (likely due to the closing of IKR by Ba<sup>2+</sup>) followed by a sodium-dependent decrease due to a tetrodotoxin-resistant Na-conductance, suggesting that both effects may occur.

The situation decribed here differs strongly from the depolarisation of  $V_{\rm m}$ , when a action potential is generated [16,18,30-33,39]. Then  $V_{\rm m}$  depolarizes due to an enormous increase in sodium conductance of the membrane. In terms of the model, IKR is dragged into the region where it closes according to its definition.

### 5. Sensitivity analysis

## 5.1. Comparison of the flux version and voltage clamp version

The statement that one may reduce the ratio q, relating the concentration changes to the electrical changes, in the ion flux version of the

model to 1% was verified by comparing the responses of the simulated intracellular concentrations and  $V_{\rm m}$  on substitutions to  $K_{\rm o}$  values of 0.1, 1, 10 and 100 mM with q=1%, 10% or 100% of the proper value. In all cases the differences were smaller than 0.3%; this was also the influence of different values of q during fast changes in the flux model (see Fig. 5). Most likely, the decrease of q does not lead to very different results because ion substitution experiments do not violate electroneutrality and all electrical sources and drains reside in the same cell membrane.

## 5.2. Variations in the definition of the kinetic elements of the model

The sensitivity of the model was investigated to variations in a number of critical parameters such as the affinity constants for Na<sub>i</sub> and  $K_o$  of the Na/K pump, the parameters in Eq. (8) describing the opening and closing of IKR and the dependence of the open channel conductance on  $K_o$  (Eq. (7)).

## 5.2.1. The sensitivity for the kinetic constants of the Na / K pump

According to Fig. 3, with the choice of  $J_{\rm p,max}$  = 150 pmol Na/cm<sup>2</sup> s and  $K_{\rm m,Na}$  = 7 mM and  $K_{\rm m,K}$  = 1 mM, the dependent variables are rather insensitive of the choice of  $J_{\rm p,max}$  and  $K_{\rm m,Na}$  for a cell at rest in control medium.

Once  $V_{\rm m}$  was steadily depolarized at  $K_{\rm o}=0.5$  mM doubling  $J_{\rm p,max}$  did not switch  $V_{\rm m}$  from a depolarization to a hyperpolarization, suggesting that under these circumstances stimulation of the pump is not very critical.

Whereas the sensitivity of the Na/K pump was taken throughout this paper as  $K_{\rm mK}=1$  mM, simulations compiled in Fig. 8 demonstrate that lowering this parameter also changes the switch-off to lower  $K_{\rm o}$ .  $K_{\rm mK}$  determines at what concentration the pump power declines (see Fig. 4d, where apparently this decline already starts at  $K_{\rm o}=2$  mM). Therefore it appears that this affinity and the sensitivity of  $V_{\rm h}$  to  $K_{\rm o}$  determine the switch-off value. With appropriate choices of  $V_{\rm h}$  and  $K_{\rm mK}$ , IKR can switch off before the pump activity declines considerably on lowering  $K_{\rm o}$  and, paradoxically,  $K_{\rm i}$  increases on reducing  $K_{\rm o}$ .

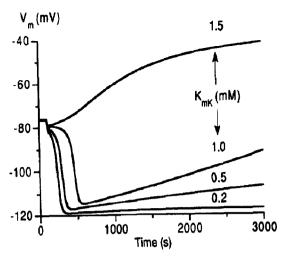


Fig. 8. The influence of the choice of  $K_{\rm mK}$ . Responses of  $V_{\rm m}$  to a reduction of  $K_{\rm o}$  to 0.7 mM at t=100 with different  $K_{\rm mK}$  values: 0.2, 0.5, 1.0 and 1.5 mM. The curve with  $K_{\rm mK}=0.2$  mM shows almost no decline, whereas the curves with  $K_{\rm mK}=0.5$  and 1.0 mM decline slowly and with  $K_{\rm mK}=1.5$  mM depolarization starts immediately.

## 5.2.2. The sensitivity for the steady state kinetics of IKR

 $V_{\rm h}$ , adapted in the simulation to fit data from the literature for skeletal muscle fibres (Eq. (8)), is only one particular solution of the model. To estimate the influence of  $V_{\rm h}$  in more general terms, the model was run with several different, and simpler, definitions for  $V_{\rm h}$ . The conclusion is that the switch-off occurred, as long as the quotient  $(V_{\rm m}-V_{\rm h})/V_{\rm s}$  became sufficiently positive. Any definition where  $V_{\rm h}$  follows  $E_{\rm K}$  in some way or another will lead to a value for  $K_{\rm o}$  where this will occur. This was checked by using of Eq. (8) the relationships  $V_{\rm h}=E_{\rm K}+30$  mV (the maximal value of Eq. (8)) and  $V_{\rm h}=E_{\rm K}-24$  mV (the minimal value of Eq. (8)): in both cases the switch-off occurred when decreasing  $K_{\rm o}$ .

The value of  $V_s$  determines the difference between  $V_m$  and  $V_h$  needed for the switch-off and the related  $K_o$  value. This value was found to decrease when  $V_s$  increases (studied were 4 mV, 6 mV, 9 mV (standard) and 13 mV, representing gating charges of about 6, 4, 3 and 2 elementary units, respectively).

Redefining  $V_h$  according to the formalism of Goldman, Hodgkin and Katz [16,28,39,41] as  $V_h$ =  $(RT/F) \ln[(K_0 + \alpha_h Na_0)/(K_i + \alpha_h Na_i)]$  allowed a more detailed examination of the dependency of the critical  $K_0$  value on  $V_h$ . The entity  $\alpha_h$  represents the selectivity of  $V_h$  for Na<sup>+</sup> with respect to  $K^+$ . Now  $V_h$  levels off at low  $K_o$ , as described normally for  $V_{\rm m}$  with constant  $P_{\rm Na}$  and  $P_{\rm K}$ . The selectivity ratio  $\alpha_{\rm h}$  in this definition may be compared with the one used to calculate  $V_{\rm m}$  $(\alpha_{\rm m} = (P_{\rm Na}/P_{\rm K})_{\rm m} = 0.01$ , at  $K_{\rm o} = 5.7$  mM, is used in the model: section 4.2). When  $\alpha_h = 0$ ,  $V_h = E_K$ and  $V_{\rm m}$  depolarized on reduction of  $K_{\rm o}$ . With the settings of the model  $\alpha_h < 0.012$  let  $V_m$  depolarize whereas  $\alpha_h > 0.013$  let it hyperpolarize, even with  $K_0 = 0.1$  mM. This last situation is no true steady state, because the Na/K pump could not cope with the loss of potassium through IKR.

## 5.3. Sensitivity of the conductance of the open IKR channel

It is clear from Fig. 6 that the similarity of the simulated  $g_K$  values and the experimental data from frog sartorius muscle is not very strong, partly because these data are not proportional to the square root of  $K_o$ . Simulations without the value  $1/\sqrt{K_o}$  in the definition of the open IKR permeability (Eq. (7)) made  $g_K$  a nearly linear function of  $K_o$ . The omission introduced quantitative differences in  $K_i$ ,  $Na_i$  and  $V_m$  as function of  $K_o$ , but it did not abolish the switch-off at sufficiently low  $K_o$ .

#### 5.4. The influence of the experimental procedure

The model suggests that the experimental method of changing the medium composition can influence considerably the outcome of the experiment and that the time needed to obtain a true steady state might be excessively long. Though Fig. 8 covers a simulation time of 50 min, runs for twice that time showed that in the cases of  $K_{\rm m,K} = 0.5$  mM and 1 mM,  $V_{\rm m}$  eventually depolarized when  $K_0 = 0.7$  mM with opened IKR due to loss of intracellular potassium and was still depolarizing at the end of the simulation. Even the data of

Fig. 4 for  $K_o$  values very near to the switch-off are still not from a true steady state. This and discrepancies in the speed of change in  $K_o$  might explain that hyperpolarized as well as depolarized  $V_{\rm m}$  values have been reported in the range of  $K_o$  around the switch-off value [1,7-10,24].

#### 6. Discussion

6.1. Inward rectification of IKR is sufficient to explain the switch-off

Section 4.5 suggests that the Na/K pump itself is not directly needed to explain the switch-off. but the disequilibrium in ion concentrations effected by it. The observed dissociation between the switch-off and loss of intracellular potassium (section 5.2.1) corroborates this conclusion. Additionally, the analysis with different formulations of the partition function (5.2.2) suggests that it occurred provided that this function becomes large enough. As long as  $V_h$  depends primarily on  $E_{\rm K}$ , such situation is always found by decreasing K<sub>0</sub> sufficiently. In all aspects IKR functions as a positive feed-back element in the cellular ionhomeostasis. When activated it stabilizes  $V_{\rm m}$ . If  $V_h$  levels off at low  $K_o$  values, and both,  $V_h$  and  $V_{\rm m}$ , can be described by a properly formulated logarithmic function (see section 5.2.2) the partition function becomes a constant. Whether this will lead to a switch-off or not depends strongly on the value of  $V_s$ . This might be why in a number of cells [7,21,52] the switch-off is not observed. It might interesting to compare the molecular structures of the IKR of such cells with those from depolarising cells.

Sections 5.3 and 5.4 suggest that even when  $V_h$  "follows"  $E_K$ , there is a range of  $K_o$  values where IKR remains open with excessive loss of intracellular potassium, with which the Na/K pump cannot cope. Kennedy et al. [14] reported that the (Na++K+)-ATPase density in the cell membrane can vary depending on  $K_o$  with a maximum in this particular range. This suggests that a continuous heavy loading of the Na/K pump may produce a regulatory signal for the incorporating more pump sites in the membrane.

6.2. Introducing more complex elementary relationships

The model presented can simulate most recordings reported earlier [10] concerning the behaviour of  $V_{\rm m}$  as function of  $K_{\rm o}$ . It is more important, that it also opens ways of investigating differences in IKR that might explain the differences generally found in such measurements [7,21,52]. For simplicity only, several relationships described in the literature have been omitted: active chloride transport [53], osmoregulation depending on the (frequently electroneutral) handling of the ions featuring in the model [44]. Also the behaviours of the two main elements in the model have been reduced to nearly caricature. One is the Na/K pump, exhibiting explicit dependence on  $V_m$  [43] and a very complex kinetic behaviour usually described by the "Post-Albers scheme" and analyzed in full detail by the late Peter Läuger [28]. The other is IKR self, demonstrating fast and transient kinetic behaviour [18,36], multi-site kinetics [33,39] (including cooperativity) and gating by intracellular magnesium [55]. All these properties were not included partly because such relationships are not yet cast in analytical forms, let it be over the total range of  $K_{\rm o}$  values presented here and for the same preparation. The results of the three previous sections nevertheless suggest that without introducing all these complexities the phenomenon of "switch-off" becomes understandable. Section 4 illustrated how the amount of computational details tends to overgrow the conceptual background of the model, though nothing, except speed of computation, will hinder their introduction.

As for muscle cells, an additional complexity may exist: the model treats the fibres as non-polar, whereas there is experimental evidence that the major portion of the conductive IKR is located in the T-system (for recent discussion, see ref. [23]). Therefore the extrusion  $K^+$  in the T-tubuli might be enhanced. Given the constricted structure of the tubuli the potassium concentration there might be slightly higher then in the medium:  $K_o$ . This would make the switch-off of the  $I_{KR}$  even more pronounced.

### 6.3. Predictive value of the model

In the section on ouabain it has been demonstrated that without an additional relationship in the model, the experimentally observed reduction in potassium selectivity cannot be understood. Changes in intracellular concentrations may provide the signal for this reduction as has been shown by including an inhibition of  $I_{KR}$  by Na<sub>i</sub>. This possibility that Na<sub>i</sub> and/or  $K_i$  modify the behaviour of IKR might also explain the spontaneous depolarization of  $V_m$  following an initial hyperpolarization in low  $K_0$ , due to Na<sub>i</sub> increase and  $K_i$  decrease during this initial hyperpolarization. Inhibition of IKR by Na<sub>i</sub> (see 4.5) has now been found experimentally [50].

In conclusion, the model can simulate the switch for hyperpolarization to depolarization, observed in a number of cell types, when  $K_0$  is lowered. It shows that this phenomenon mainly depends on the sensitivity of the open/close partition function of the inward rectifying potassium channel to the electrochemical potassium gradient across the cell membrane. This sensitivity can be described by a Boltzmann distribution function with the parameters  $V_{\rm h}$  (the membrane potential when the partition function is half maximal) and  $V_s$  ("the slope" quantifying the speed of change in the function [28]).  $V_h$  can be described with simple as well as complex equations (like Eq. (8) in this paper) without losing the aspects of the general observation.

#### Acknowledgement

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